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Simple additive weighting—A metamodel for multiple criteria decision analysis methods

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ABSTRACT

Multiple Criteria Decision Analysis methods, such as ELECTRE, PROMETEE, AHP, TOPSIS, VIKOR, have been applied to solving numerous real-life decision making problems in business and management. However, the mechanics of those methods is not easily understandable and it is often seen by users without much formal training as a kind of "scientific witchcraft".

In order to make those popular MCDA methods more transparent, we provide a simple framework for interpretations of rankings they produce. The framework builds on the classical results of MCDA, in particular on the preference capture mechanism proposed by Zionts and Wallenius in seventies of the last century, based on Simple Additive Weighting.

The essence and the potential impact of our contribution is that given a ranking produced by a MCDM method, we show how to derive weights for the Simple Additive Weighting which yield the same ranking as the given method. In that way we establish a common framework for almost no–cost posterior analysis, interpretation and comparison of rankings produced by MCDA methods in the expert systems environment. We show the working of the concept taking the TOPSIS method in focus, but it applies in the same way to any other MCDM method.

We illustrate our reasoning with numerical examples taken from literature.

1. Introduction

Multiple Criteria Decision Analysis (MCDA) is a topic well represented in the field of expert systems; many of them employ MCDA for solving complex problems of decision making (see Alemi-Ardakani, Milani, Yannacopoulos, & Shokouhi, 2016; Mardani, Jusoh, & Zavadskas, 2015; Östermark & Salmela, 1988; Ozernoy, 1988).

Solving an MCDA problem is usually understood as determining an alternative (a decision variant) which corresponds to the best, in the decision maker's opinion, combination of (at least two) criteria values, or in a broader sense, as ranking alternatives from the best (in the above meaning) to the worst one. Because attaining the maximal values with respect to all criteria simultaneously, in general, impossible, solving an MCDA problem requires that some information on preferred combinations of criteria values (so called *preference information*) has to be articulated by the *decision maker* (DM).

The representative and the most popular methods of MCDA within the expert systems domain are ELECTRE (Roy, 1968), PROMETHEE (Brans & Vincke, 1985), AHP (Saaty, 1990), TOPSIS (Hwang & Yoon, 1981), VIKOR (Opricovic & Tzeng, 2004), see e.g. Özcan, Çelebi, and Esnaf 2011; Tsou 2008. In the MCDA methods, preference information, coming from the DM, is fed into some selection mechanism and then this mechanism, representing (or just *usurping* to represent) the DM's *preference model*, is used to derive the preferred ranking.[1] Although the DM is free in providing preference information, the resulting ranking is conditioned not only by the DM's preferences but also by the properties of the method mechanism. Then the following question arises: *Does a MCDA method mechanisms represent correctly the DM's preferences?*

Due to its conceptual and implementation simplicity, the MCDA methods are available to a wide range of practitioners who are not necessary experts in MCDA and thus they cannot answer questions of that sort by themselves, neither a priori nor a posteriori.

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[1] Thus, according to the taxonomy given by Miettinen (1999) these methods belong to the class of methods with *a priori* articulation of preference information, where methods with *a posteriori* and *interactive* articulation constitute the two other classes.

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However, there is a lack of a methodology which would enable to understand and interpret the results of all those methods in a uniform way. The mechanics of the MCDA methods, such as ELECTRE, PROMETEE, AHP, TOPSIS or VICKOR, are not easily interpretable and therefore their working can be seen by users without much formal training as a kind of "scientific witchcraft".

So in order to make the MCDA methods more transparent, we propose the firmly established and with no doubt the most widely known Single Average Weighting method to play the role of a metamodel. We show how to interpret the MCDA methods in the terms of the metamodel. We build the interpretation on the classical results of MCDA, namely on the preference capture mechanism – the Simple Additive Weighting (SAW) – proposed by Zionts and Wallenius (1983). The framework we propose can help to reinterpret and thus to *demystify* the results an MCDA method provides. We show its working using the TOPSIS method as an example, but ELECTRE, PROMETEE, AHP, VIKOR, and virtually any MCDA method, can be interpreted and reinterpreted in the analogous way.

There have been studies concerning significance, derivation and interpretation of weights in MCDA. For example, Alemi-Ardakani et al. (2016) analyzed how different subjective and objective weighting methods capture preferences in terms of ranking of alternatives. For other studies one can point to the works of Deng, Yeh, and Willis (2000); Figueira and Roy (2002); Kung and Wen (2007); Olson (2004); Rogers and Bruen (1998). All those studies concentrate on theoretical analysis of weighting and providing general recommendations. The novelty of our approach is twofold. First, it falls into the reverse engineering paradigm (Kaliszewski, 2016), where starting from a ranking of alternatives, the set of weight resulting in such a ranking is derived. Second, our approach provides a practical tool which can be applied to any real–life decision making problem, whatever MCDA method was used. Indeed, as shown below, the input of our approach is just the ranking produced by a MCDA method.

This paper is organized as follows. In the next section we recall the TOPSIS method formulation.

In Section 3, we point to some elements in the TOPSIS method which seem lacking a methodological basis and are arbitrary.

In Section 4 we propose a simple explanatory framework to interpret the rankings the method provides an easy terms of *linear weighting functions*, in other words, in terms of SAW.

To illustrate our concept, in Section 5 we solve the numerical problem, given in the original paper by Hwang and Yoon, by the TOPSIS method and we interpret the resulting ranking in terms of that framework. Section 6 concludes.

2. The TOPSIS method

TOPSIS, a ranking method in the field of Multiple Criteria Decision Analysis (MCDA), was developed by Chen and Hwang (1992); Hwang and Yoon (1981). It has become very popular among practitioners, first in Asia-Pacific region and then across the world. In the paper (Behzadian, Otaghsara, Yazdani, & Ignatius, 2012) 266 journal papers are collected, published since 2000 on TOPSIS applications in such areas as Supply Chain Management; Engineering Design, Engineering and Manufacturing Systems; Business and Marketing Management, and others. TOPSIS has been also extended for group decision making (for a survey of works in that directions see e.g. Huang & Li, 2012).

In the TOPSIS method, preference information, coming from the DM in the form of criteria weights, is fed into a parametric function (the *TOPSIS valuation function*) and then this function, representing the DM's *preference model*, is used to derive the preferred ranking. Although the DM is free in providing preference information — weights, the resulting ranking is conditioned not only by weights but also by the properties of the TOPSIS valuation function. Then the following question arises: *Does the TOPSIS valuation function represent correctly the DM's preferences?*

The MCDA problem the TOPSIS method can handle is framed as follows. Given are the *decision matrix* $D = \{d_{ij}\}$, with the alternatives A_i , $i = 1, \dots, m$, (rows), and the criteria X_j , $j = 1, \dots, k$, (columns). To simplify the presentation but without loss of generality, we assume that all criteria are of the type "the more the better".

The TOPSIS method ranks alternatives from the "best" to "worst" and to do that it uses solely criteria values and criteria weights w_j , $j = 1, \dots, k$, which are given by the DM.

To work with comparable magnitudes of numbers, the first step in TOPSIS is to normalize criteria values. The issue of normalization in TOPSIS is well covered in the literature (see e.g. Milani 2005) Q3 and therefore we skip that topic here. Below we shall use the normalization with the Euclidean norm, by Hwang and Yoon (1981).

The TOPSIS method consists of the following steps.

1. Given the decision matrix D, calculate the normalized decision matrix $R = \{r_{ij}\}$,
   $r_{ij} = d_{ij}/\sqrt{\sum_i d_{ij}^2}$, $i = 1, \dots, m$, $j = 1, \dots, k$.
2. Calculate the weighted decision matrix $V = \{v_{ij}\}$,
   $v_{ij} = w_j r_{ij}$, $i = 1, \dots, m$, $j = 1, \dots, k$.
3. Determine the ideal solution A^* and the negative-ideal solution A^- ,
   $A^*_j = \max_i v_{ij}$, $j = 1, \dots, k$, $A^-_j = \min_i v_{ij}$, $j = 1, \dots, k$.
4. For each alternative $i = 1, \dots, m$, calculate the separation measures S^*_i and S^-_i ,
   $S^*_i = \sqrt{\sum_j (v_{ij} - A^*_j)^2}$, $S^-_i = \sqrt{\sum_j (v_{ij} - A^-_j)^2}$.
5. For each alternative $i = 1, \dots, m$, calculate the relative closeness C_i to the ideal solution,
   $C_i = S^-_i / (S^-_i + S^*_i)$.
6. Rank alternatives with the descending order of C_i .

In the next section we address the TOPSIS–specific weight mechanics and rank reversal issues.

3. TOPSIS mysteries

Let $a = (a_1, a_2, \dots, a_k)$ denote any point of the box A defined by A^*_j and A^-_j , $j = 1, \dots, k$.

From the description of the TOPSIS method it follows that the TOPSIS preference model consists of weight–dependent valuation function C_a,

$$C_a = \frac{S^-_a}{S^-_a + S_a*}$$

(see Step 5 in the TOPSIS description in the previous section), and weights w_j, $j = 1, \dots, k$, given by the DM. Because of the specific form of the valuation function C_a its usage raises some concern and calls for the utmost consciousness.

Any ranking process consists of two parts: the cognitive part— elicitation of the preference model — and the technical part— calculation of the ranking. TOPSIS realizes just the technical part of the ranking process, taking weighs w_j and decision matrix D as the input. In other words, in TOPSIS the cognitive part is absent.

The main concern in TOPSIS is about how is the input (weights) related to output (the resulting ranking)?

As can be seen from the description given above, in TOPSIS weights do not weight criteria directly. Indeed, in the separation measures S^*_a and S^-_a one has

$$S^*_a = \sqrt{\sum_j w_j^2 (a_j - A^*_j)^2} ,$$

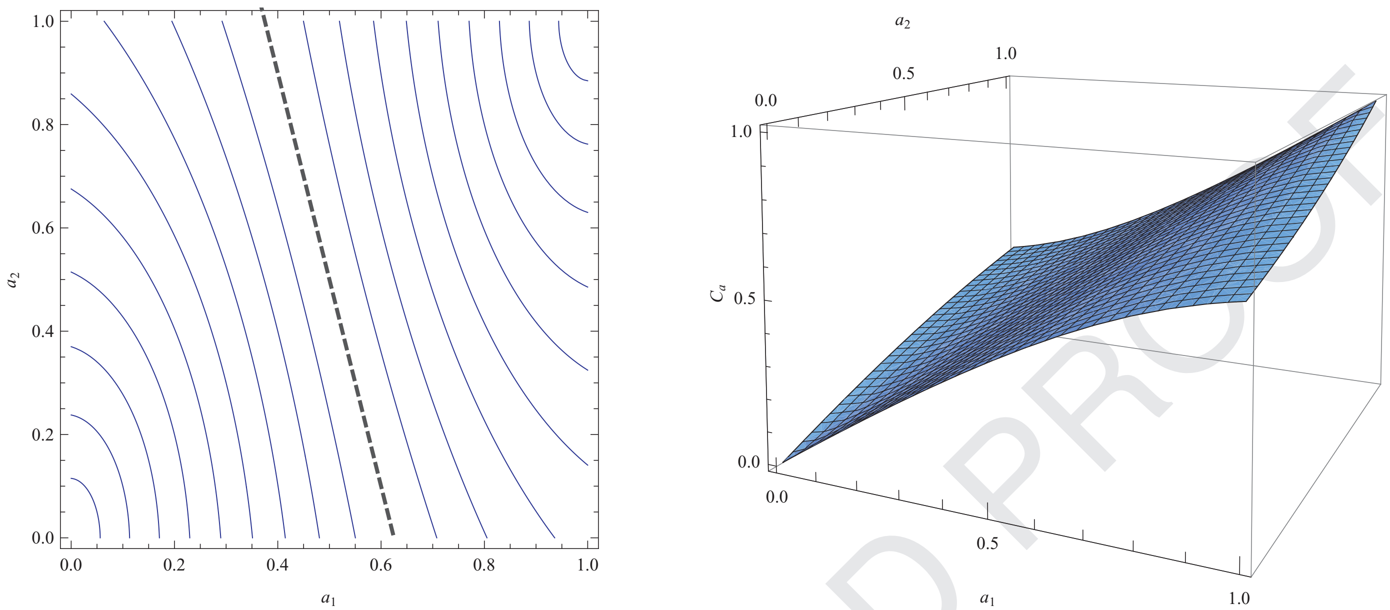

Fig. 1. Contour plot (left) and 3D plot (right) of function C_a for $A_j^* = 1,\ A_j^- = 0,\ \ j = 1, \ldots, k,\ \ w_1 = 2,\ \ w_2 = 1$.

and

$$S_a^- = \sqrt{\sum_j w_j^2 (a_j - A_j^-)^2}\ .$$

Here the role and purpose of weights is far from being intuitive. For example, Fig. 1 presents contours of C_a for the case $w_1 = 2,\ w_2 = 1$, where for the sake of simplicity $A_i^* = 1,\ A^- = 0,\ i = 1, \ldots, k$. The middle contour is linear and corresponds to the line $4a_1 + a_2 = b$, where b is a constant. In other words, along this contour alternatives are valuated as in the weighted sum function $w_1 a_1 + w_2 a_2$ with $w_1 = 4,\ w_2 = 1$, not necessarily to the DM intention and definitely behind its conscious choice.

One may ask why not to apply weights directly to the decision matrix D to get matrix $V' = \{v'_{ij}\}$, where $v'_{ij} = w_i d_{ij}$, and only then normalize matrix V'. However, such an action would result in the non–weighted normalized decision matrix R, since

$$\frac{v'_{ij}}{\sqrt{\sum_i (v'_{ij})^2}} = \frac{w_i d_{ij}}{\sqrt{\sum_i (w_i d_{ij})^2}} = \frac{d_{ij}}{\sqrt{\sum_i (d_{ij})^2}} = r_{ij}.$$

So application of weights to the normalized matrix R is the only viable option. But this action results in matrix V whose column norms are not, in general, equal to one. Norm values in matrix V are now equal to weights, which is the immediate result of multiplying a normalized vector by a scalar (here weight). As weights can easily differ by the factor of 10, the original idea to work with data of the same magnitude is clearly lost.

Finding no hints in the TOPSIS method for its easy interpretations, below we proposes a simple explanatory framework which, given the ranking resulting from the TOPSIS method, can help the DM to understand and eventually accept or discard that ranking.

Although our starting point was the TOPSIS method, from the exposition given below it is quite clear that our arguments apply to any MCDA method, with TOPSIS serving as an example.

4. TOPSIS demystified

To simplify the presentation and without loss of generality, in this section we assume that m alternatives $A_i,\ \ i = 1, \ldots, m$, represented by the normalized matrix R, are ranked by the TOPSIS method as $A_1 \succ \cdots \succ A_m$, with the first alternative A_1 and no two alternatives having the same rank. Below, in a numerical example we consider the case where the TOPSIS ranking is equivocal.

One can ask: do there exist weights of the weighted linear function (as in the Simple Additive Weighting approach) over normalized values of k criteria,

$$w_1 r_{*,1} + \cdots\ + w_k r_{*,k}\ , \tag{1}$$

which guarantee the same ranking (resulting from sorting the alternatives A_i in the descending order of the value of this function) as in TOPSIS? If yes, what they are? If not, are there other functions which guarantee the same ranking as in TOPSIS?

To guarantee ranking $A_1 \succ \cdots \succ A_m$, weights have to satisfy the following system of $(m-1)+k$ linear inequalities and 1 equation

$$\begin{aligned} \sum_{j=1}^k w_j r_{ij} \geq \sum_{j=1}^k w_j r_{i+1,j}\,, \quad i = 1, \ldots, m-1\,, \\ w_j \geq 0\,, \quad j = 1, \ldots, k\,, \\ \sum_{j=1}^k w_j = 1\,. \end{aligned} \tag{2}$$

The last constraint is imposed to avoid the trivial solution $w_j = 0,\ \ j = 1, \ldots, k$. Actually, instead of 1 as the right hand side value any positive number can be used here, but putting 1 is customary and convenient.

The above condition is the immediate consequence of the original idea of Zionts and Wallenius to capture the DM's preference by means of weights of function (1) (cf. Zionts & Wallenius, 1983).

Let us denote the set of weights satisfying (2) by W. If W is nonempty and not a singleton, then there is an infinite number of weight vectors which under the weighted linear function yield the same ranking as the TOPSIS ranking. In other words, for any element from set W, TOPSIS and SAW produce the same ranking.

Example 1 - the set W contains an infinite number of elements.

Consider the ranking problem with matrix D and weights as given in Table 1.

The TOPSIS method produces the following (unequivocal) ranking: $A_2 \succ A_3 \succ A_4 \succ A_1$.

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Table 1
Example 1 – the decision matrix D and the (TOPSIS) weights.

	X_1	X_2
A_1	2.0	1500
A_2	2.5	2700
A_3	1.8	2000
A_4	2.2	1800
w_i	0.5	0.5

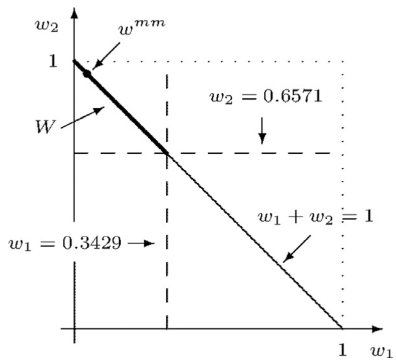

Fig. 2. The set W in Example 1 (the thick line segment of $w_1 + w_2 = 1$); w^{mm} denotes the middle most element of W, as explained at the end of this Section.

In this problem set W is defined by the following set of 5 linear inequalities and 1 equation.

$$\begin{array}{ll} w_1\,0.5839 + w_2\,0.6591 \geq w_1\,0.4204 + w_2\,0.4882 & (\text{which stands for } A_2 \succ A_3)\,, \\ w_1\,0.4204 + w_2\,0.4882 \geq w_1\,0.5139 + w_2\,0.4394 & (\text{which stands for } A_3 \succ A_4)\,, \\ w_1\,0.5139 + w_2\,0.4394 \geq w_1\,0.4671 + w_2\,0.3662 & (\text{which stands for } A_4 \succ A_1)\,, \\ w_1 \geq 0\,, & \\ w_2 \geq 0\,, & \\ w_1 + w_2 = 1\,. & \end{array} \tag{3}$$

After regrouping the terms we have

$$\begin{array}{r} w_1\,0.1635 + w_2\,0.1709 \geq 0\,, \\ -w_1\,0.0935 + w_2\,0.0488 \geq 0\,, \\ w_1\,0.0468 + w_2\,0.0732 \geq 0\,, \\ w_1 \geq 0\,, \\ w_2 \geq 0\,, \\ w_1 + w_2 = 1\,. \end{array} \tag{4}$$

Putting $w_1 = 1 - w_2$ we have

$$0.6571 \leq w_2 \leq 1\,, \tag{5}$$

and from this we obtain

$$0 \leq w_1 \leq 0.3429\,. \tag{6}$$

The set W for this Example is represented in Fig. 2.

End of example 1

Example 2 - the set W is empty.

Consider the ranking problem with matrix D and weights as given in Table 2.

The TOPSIS method produces the following (equivocal) ranking: $A_3 \succ A_1 \equiv A_2 \succ A_4$, where $A_1 \equiv A_2$ denotes that A_1 and A_2 are ranked analogously (as second in the ranking).

In this problem, set W is defined by the following set of 6 linear inequalities and 1 equation (we do not require the condition $A_1 \equiv A_2$ to hold).

$$\begin{array}{ll} w_1\,0.5268 + w_2\,0.5268 \geq w_1\,0.3710 + w_2\,0.7419 & (\text{which stands for } A_3 \succ A_1)\,, \\ w_1\,0.5268 + w_2\,0.5268 \geq w_1\,0.7419 + w_2\,0.3710 & (\text{which stands for } A_3 \succ A_2)\,, \\ w_1\,0.3710 + w_2\,0.7419 \geq w_1\,0.1855 + w_2\,0.1855 & (\text{which stands for } A_1 \succ A_4)\,, \\ w_1\,0.7419 + w_2\,0.3710 \geq w_1\,0.1855 + w_2\,0.1855 & (\text{which stands for } A_2 \succ A_4)\,, \\ w_1 \geq 0\,, & \\ w_2 \geq 0\,, & \\ w_1 + w_2 = 1\,. & \end{array} \tag{7}$$

Table 2
Example 2 – the decision matrix D and the (TOPSIS) weights.

	X_1	X_2
A_1	0.5	1.0
A_2	1.0	0.5
A_3	0.71	0.71
A_4	0.25	0.25
w_i	0.5	0.5

After regrouping the terms we have

$$\begin{array}{r} w_1\,0.1558 - w_2\,0.2151 \geq 0\,, \\ -w_1\,0.2151 + w_2\,0.1558 \geq 0\,, \\ w_1\,0.1855 + w_2\,0.5564 \geq 0\,, \\ w_1\,0.5564 + w_2\,0.1855 \geq 0\,, \\ w_1 \geq 0\,, \\ w_2 \geq 0\,, \\ w_1 + w_2 = 1\,. \end{array} \tag{8}$$

Putting $w_1 = 1 - w_2$ we have

$$\begin{array}{l} 0 \leq w_2 \leq 0.4201\,, \\ 0.5799 \leq w_2 \leq 1\,. \end{array} \tag{9}$$

and as these conditions are inconsistent, hence W is empty.

However, using the quadratic weighted function over an alternative criteria values

$$(w_1 r_{*,1})^2 + \cdots + (w_k r_{*,k})^2\,, \tag{10}$$

and solving the following set of 6 linear inequalities and 1 equation (we do not require the condition $A_1 \equiv A_2$ to hold)

$$\begin{array}{ll} (w_1\,0.5268)^2 + (w_2\,0.5268)^2 \geq (w_1\,0.3710)^2 + (w_2\,0.7419)^2 & (\text{which stands for } A_3 \succ A_1)\,, \\ (w_1\,0.5268)^2 + (w_2\,0.5268)^2 \geq (w_1\,0.7419)^2 + (w_2\,0.3710)^2 & (\text{which stands for } A_3 \succ A_2)\,, \\ (w_1\,0.3710)^2 + (w_2\,0.7419)^2 \geq (w_1\,0.1855)^2 + (w_2\,0.1855)^2 & (\text{which stands for } A_1 \succ A_4)\,, \\ (w_1\,0.7419)^2 + (w_2\,0.3710)^2 \geq (w_1\,0.1855)^2 + (w_2\,0.1855)^2 & (\text{which stands for } A_2 \succ A_4)\,, \\ w_1 \geq 0\,, & \\ w_2 \geq 0\,, & \\ w_1 + w_2 = 1\,, & \end{array} \tag{11}$$

one gets feasible weights, for example $w_1 = 0.0400,\ w_2 = 0.9600$. In other words, the above system of conditions is consistent.

End of example 2

Example 2 shows that one can attempt to explain the TOPSIS rankings also by other functions than the linear weighted function. Clearly though, the linear weighted function is the simplest and easiest to use and interpret.

If the number of weights in W is infinite, whatever is the function used to explain the TOPSIS rankings, the DM may need some assistance in analyzing and understanding set W. For example the DM can ask a series of questions about the ranges of weight variations preserving the TOPSIS ranking.

Here are examples of such questions, all pertaining to preservation of the TOPSIS ranking.

1. What is the range of individual weight variations?

2. Taking the weights as in the TOPSIS ranking, what are other weight contributions preserving the TOPSIS ranking?

3. What weights are the middle most, where the middle most weights are defined as an element of W the distance from which to all boundaries of W is the greatest?[2] [3]

[2] The middle most weights offer the greatest stability of the TOPSIS ranking with respect to possible weight perturbations within the SAW model.

[3] The middle most weight can be found by solving the following optimization problem

$$\begin{array}{l} \max\ t \\ \sum_{j=1}^{k} w_j (r_{ij} - r_{i+1,j}) \geq t\,, \quad i = 1, \ldots, m-1\,, \\ w_j \geq t\,, \quad j = 1, \ldots, k\,. \end{array} \tag{12}$$

see Chmielewski and Kaliszewski (2011).

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Table 3
Hwang and Yoon problem – the decision matrix D and the (TOPSIS) weights.

	X_1	X_2	X_3	X_4	X_5	X_6
A_1	2.0	1500	20000	-5.5	5	9
A_2	2.5	2700	18000	-6.5	3	5
A_3	1.8	2000	21000	-4.5	7	7
A_4	2.2	1800	20000	-5.0	5	5
w_i	0.2	0.1	0.1	-0.1	0.2	0.3

4. Do TOPSIS weights preserve the TOPSIS ranking when applied to the linear function?

5. Do equal weights preserve the TOPSIS ranking?

There can be cases in which one can only ask about weights which preserve a part of the TOPSIS ranking, e.g. only at a few first positions or just preserve the first position. In such cases the set of inequalities (2) should be reduced accordingly.

5. Hwang and Yoon example

We shall illustrate our argument by the problem presented by Hwang and Yoon in their original exposition of TOPSIS (Hwang & Yoon, 1981); the results are given in Table 4.

The decision matrix D and the weights are given in Table 3.[4]

The TOPSIS method produces the following (unequivocal) ranking: $A_1 \succ A_3 \succ A_4 \succ A_2$.

Calculations for Table 4 were done in Excel add–in *Solver*, with the LP simplex as the solving method.

Analyzing Table 4, the DM may note for example that for all criteria excluding the sixth one, minimum weights are zero. This means that any of these criteria can be ignored without losing information necessary for ranking. Other conclusions can be drawn from comparison of TOPSIS weights and maximum weights. For example, it is possible to assign the third criterion as big weight as 0.855 having the same ranking as TOPSIS yields in the case of this weight equal to 0.1. Analyzing lines from 13th to 19th, the DM can grasp an idea about coefficients of substitutions among criteria, which can be used for justifying the decision.

6. Concluding remarks

The MCDA methods are widely used in expert systems literature but they lack transparency of their decision rules. By lack of transparency we mean that their working escapes an easy grasp by non–specialists, individuals from outside of the decision making domain. And the transparency of decision making/decision support expert systems is absolutely indispensable if they are to become practical tools assisting real DMs in their daily tasks.

Ranking by MCDA methods is, as a rule, one–iteration process. This does not conform well with nowadays broadly accepted paradigm that decision processes are by their very nature interactive (Simon, 1977; for interactive ranking model building and interactive ranking selection see Chmielewski & Kaliszewski, 2011). In one–iteration processes the DM has no chance to learn about the interplay of criteria by observing the results of his trial decisions. On the contrary, his/her preferences, not necessary sharp and definite, are irrevocably applied to a decision selection mechanism.

The rigid and somewhat obscure (to non–specialists) framework of MCDA methods can be relaxed a bit by adding to each of them a kind of *posterior* analysis, leaving however the use of those methods intact, as proposed in this paper. Irrespective of the MCDA method selected, SAW, as we proposed the use of it in the paper, offers a level ground to interpret and compare (if more than one method has been selected to do the job) the results – rankings of alternatives. By our development we allow the DM to interpret the MCDA method–produced rankings in the much simpler terms than the original MCDA methods allow, in the almost natural language of SAW (SAW belongs nowadays to the folklore of the domains of

[4] Since the fourth criterion was of the type "the less the better", to satisfy our assumption that all criteria are of the type "the more the better", we were to multiply all the values of that criterion by −1.

Table 4
Hwang and Yoon problem – the analysis of the TOPSIS ranking by the linear weighting function. To enable selection of weights in calculations for line 13, w_2 was maximized, whereas for lines 14–18, w_1 was maximized.

	Question no.		w_1	w_2	w_3	w_4	w_5	w_6
1	1	min w_1	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
2	1	max w_1	0.6576	0.0000	0.0000	0.0000	0.2394	0.1030
3	1	min w_2	0.0000	0.0000	0.1236	0.4677	0.0431	0.3657
4	1	max w_2	0.0000	0.2277	0.0000	0.3620	0.0000	0.4103
5	1	min w_3	0.0000	0.2277	0.0000	0.3620	0.0000	0.4103
6	1	max w_3	0.0000	0.0000	0.8550	0.0000	0.0000	0.1450
7	1	min w_4	0.0000	0.0000	0.8550	0.0000	0.0000	0.1450
8	1	max w_4	0.0000	0.0000	0.0000	0.6180	0.0000	0.3820
9	1	min w_5	0.0000	0.0000	0.0000	0.6180	0.0000	0.3820
10	1	max w_5	0.0000	0.0000	0.0000	0.0000	0.4365	0.5635
11	1	min w_6	0.2545	0.0000	0.7056	0.0000	0.0000	0.0399
12	1	max w_6	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
13	2	w_1 as for TOPSIS	0.2000	0.1591	0.0000	0.3543	0.0000	0.2866
14	2	w_2 as for TOPSIS	0.3722	0.1000	0.0000	0.3476	0.0000	0.1801
15	2	w_3 as for TOPSIS	0.6004	0.0000	0.1000	0.0000	0.2055	0.0941
16	2	w_4 as for TOPSIS	0.6053	0.0000	0.0000	0.1000	0.1845	0.1103
17	2	w_5 as for TOPSIS	0.6326	0.0000	0.0000	0.0422	0.2000	0.1252
18	2	w_6 as for TOPSIS	0.5132	0.0000	0.0000	0.0000	0.1868	0.3000
19	3	mid. most w_i	0.0401	0.0401	0.0401	0.3038	0.0401	0.5358
20	4	all w_i as for TOPSIS			Ranking A_4, A_3, A_2, A_1.			
21	5	all $w_i = 0, 1667$			Ranking A_4, A_3, A_2, A_1.			

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decision making and expert systems, and hardly needs much explanation).

It is noteworthy to observe that the set W produced in our approach can be interpreted as an archive of weights, all of them leading to the same ranking. Such archives can serve for a kind of *knowledge discovery* about the interplay of weights and criteria with a significantly simpler preference model than that offered by MCDA methods. Moreover, such archives can be a spring off point to relate MCDA methodologies to other decision making concepts as *case–based reasoning* (see e.g. Bergmann et al., 2003) or *multi–step* (iterative) *decision processes* (in the sense of Simon, 1977).

Uncited references

García-Cascales and Lamata (2012); Lai, Liu, and Hwang (1994); Roy (1991); Shih, Shyur, and Lee (2007); Tzeng, Lina, and Opricovic (2005); Wierzbicki (2007).

Appendix

We repeat calculations as for Hwang and Yoon example in Section 5 for the problem taken from the work of Goh, Tung, and Cheng (1996); the results are given in Table 6. This problem in the original paper was not solved by TOPSIS but by the other method.

As before, weight related calculations were done in Excel add–in *Solver*, with the LP simplex as the solving method.

The decision matrix D and weights are given in Table 5.

The TOPSIS method produces the following (unequivocal) ranking: $A_4 \succ A_3 \succ A_2 \succ A_1$.

Table 5
Goh, Tung and Cheng problem – the decision matrix D and the weights.

	X_1	X_2	X_3	X_4	X_5	X_6
A_1	0.3750	0.3000	0.1401	0.1290	0.3200	0.1818
A_2	0.2916	0.2667	0.2293	0.1935	0.2800	0.2273
A_3	0.1667	0.2333	0.3153	0.2903	0.2400	0.2727
A_4	0.1667	0.2000	0.3153	0.3872	0.1600	0.3182
w_i	0.1860	0.1860	0.1396	0.1396	0.1860	0.1628

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Table 6
Goh, Tung and Cheng problem – the analysis of the TOPSIS ranking by the linear weighting function. To enable selection in calculations for line 13, w_2 was maximized, whereas for lines 14–18, w_1 was maximized.

	Question no.		w_1	w_2	w_3	w_4	w_5	w_6
1	1	min w_1	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
2	1	max w_1	0.4335	0.0000	0.0024	0.5641	0.0000	0.0000
3	1	min w_2	0.4331	0.0000	0.0000	0.5669	0.0000	0.0000
4	1	max w_2	0.0000	0.6695	0.0861	0.2444	0.0000	0.4103
5	1	min w_3	0.4331	0.0000	0.0000	0.5669	0.0000	0.0000
6	1	max w_3	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
7	1	min w_4	0.4122	0.0000	0,5878	0.0000	0.0000	0.0000
8	1	max w_4	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
9	1	min w_5	0.4331	0.0000	0.0000	0.5669	0.0000	0.0000
10	1	max w_5	0.0000	0.0000	0.0000	0.4632	0.5368	0.0000
11	1	min w_6	0.4331	0.0000	0.0000	0.5669	0.0000	0.0000
12	1	max w_6	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
13	2	w_1 as for TOPSIS	0.1860	0.4089	0.1468	0.2583	0.0000	0.0000
14	2	w_2 as for TOPSIS	0.3209	0.1860	0.0681	0.4250	0.0000	0.0000
15	2	w_3 as for TOPSIS	0.4285	0.0000	0.1396	0.4319	0.0000	0.0000
16	2	w_4 as for TOPSIS	0.4175	0.0000	0.4429	0.1396	0.0000	0.0000
17	2	w_5 as for TOPSIS	0.3152	0.0000	0.0806	0.4182	0.1860	0.0000
18	2	w_6 as for TOPSIS	0.0772	0.0772	0.2065	0.4846	0.0772	0.0772
19	3	mid. most w_i	0.0000	0.0000	0.2603	0.7397	0.0000	0.0000
20	4	All w_i as for TOPSIS			Ranking A_1, A_2, A_3, A_4.			
21	5	all $w_i = 0.1667$			Ranking A_3, A_1, A_4, A_2.			

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